

ELEC 421 – Assignment # 4

1. The Laplacian operator for normal/unrotated coordinates is defined as

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

and by the following equation for rotated coordinates

$$\nabla^2 f = \frac{\partial^2 f}{\partial x'^2} + \frac{\partial^2 f}{\partial y'^2}$$

It is given that the relationship of coordinates for a rotation by angle θ is given by

$$\begin{aligned}x &= x' \cos \theta - y' \sin \theta \\y &= x' \sin \theta + y' \cos \theta\end{aligned}$$

where (x,y) are the unrotated and (x',y') are the rotated coordinates.

Show that the Laplacian as defined above is invariant to rotation.

2. Assume an average 3x3 filter that uses the four closest neighbours and excludes the center point from the average.
- Determine the filter in the frequency domain.
 - Is this a high-pass or a low-pass filter? Explain your answer.